

VARIABLE MOTION (PART 2)

(1) x-direction	y-direction
$a_x = 4t$	$y = 8t^3 - 6t$
Integrating	Diff w.r.t t
$v_x = \frac{4t^2}{2} + c$	$v_y = 24t^2 - 6$
	Diff w.r.t t
At $t=1, v_x = 6 \text{ cm/s}$	$a_y = 48t$
$\therefore c = 4$	
$\therefore v_x = 2t^2 + 4$	
$\therefore a = 4t \hat{i} + 48t \hat{j}$	
$\bar{a} _{t=3} = 12 \hat{i} + 144 \hat{j}$	
$\therefore a _{t=3} = \sqrt{12^2 + 144^2} = 144.5 \text{ m/s}^2$	
$\bar{v} = (2t^2 + 4) \hat{i} + (24t^2 - 6) \hat{j}$	
$\bar{v} _{t=3} = 22 \hat{i} + 210 \hat{j}$	
$v _{t=3} = \sqrt{22^2 + 210^2} = 211.5 \text{ m/s}$	

(2) x-direction	y-direction	z-direction
$a_x = 3t$	$a_y = 12t^2$	$a_z = 5$
Integrating	Integrating	Integrating
$v_x = \frac{3t^2}{2} + c$	$v_y = 4t^3 + c$	$v_z = 5t + c$
Put $t=0, v_x=0$	Put $t=0, v_y=0$	Put $t=0, v_z=0$
$\therefore c=0$	$\therefore c=0$	$\therefore c=0$
$v_x = \frac{3t^2}{2}$	$v_y = 4t^3$	$v_z = 5t$
Integrating	Integrating	Integrating
$x = \frac{t^3}{2} + c$	$y = t^4 + c$	$z = \frac{5t^2}{2} + c$
Put $t=0, x=-5$	Put $t=0, y=-2$	Put $t=0, z=4$
$\therefore c=-5$	$\therefore c=-2$	$\therefore c=4$
$x = \frac{t^3}{2} - 5$	$y = t^4 - 2$	$z = \frac{5t^2}{2} + 4$

$\therefore \bar{a} = 3t \hat{i} + 12t^2 \hat{j} + 5 \hat{k}$
 $\bar{a}|_{t=2} = 6 \hat{i} + 48 \hat{j} + 5 \hat{k}$
 $a|_{t=2} = \sqrt{6^2 + 48^2 + 5^2}$
 $= 48.63 \text{ m/s}^2$

$\bar{v} = \frac{3t^2}{2} \hat{i} + 4t^3 \hat{j} + 5t \hat{k}$
 $\bar{v}|_{t=2} = 6 \hat{i} + 32 \hat{j} + 10 \hat{k}$
 $v|_{t=2} = \sqrt{6^2 + 32^2 + 10^2}$
 $= 34.06 \text{ m/s}$

$\bar{r} = (\frac{t^3}{2} - 5) \hat{i} + (t^4 - 2) \hat{j} + (\frac{5t^2}{2} + 4) \hat{k}$
 $\bar{r}|_{t=2} = -1 \hat{i} + 14 \hat{j} + 14 \hat{k}$
 $\therefore r|_{t=2} = \sqrt{1^2 + 14^2 + 14^2}$
 $= 19.82 \text{ m}$

displacement = $(-1 - (-5)) \hat{i}$
 $+ (14 - (-2)) \hat{j}$
 $+ (14 - 4) \hat{k}$
 $= 4 \hat{i} + 16 \hat{j} + 10 \hat{k}$
 $\therefore \text{displacement} = \sqrt{4^2 + 16^2 + 10^2}$
 $= 19.29 \text{ m}$



(3)

x direction	y direction	z direction
$a_x = 12t^2$	$a_y = -16t$	$a_z = 8$
Integrating	Integrating	Integrating
$v_x = 4t^3 + c$	$v_y = -8t^2 + c$	$v_z = 8t + c$
At $t=0, v_x=4$	at $t=0, v_y=-3$	at $t=0, v_z=2$
$\therefore c=4$	$\therefore c=-3$	$\therefore c=2$
$\therefore v_x = 4t^3 + 4$	$v_y = -8t^2 - 3$	$v_z = 8t + 2$
Integrating	Integrating	Integrating
$x = t^4 + 4t + c$	$y = -\frac{8t^3}{3} - 3t + c$	$z = 4t^2 + 2t + c$
At $t=0, x=16$	at $t=0, y=-20$	at $t=0, z=5$
$\therefore c=16$	$\therefore c=-20$	$\therefore c=5$
$x = t^4 + 4t + 16$	$y = -\frac{8t^3}{3} - 3t - 20$	$z = 4t^2 + 2t + 5$

$\vec{a} = 12t^2 \hat{i} - 16t \hat{j} + 8 \hat{k}$
 $\vec{a}|_{t=5} = 300 \hat{i} - 80 \hat{j} + 8 \hat{k}$
 $a|_{t=5} = \sqrt{300^2 + 80^2 + 8^2} = 310.59 \text{ m/s}^2$
 $\vec{v} = (4t^3 + 4) \hat{i} + (-8t^2 - 3) \hat{j} + (8t + 2) \hat{k}$
 $\vec{v}|_{t=5} = 504 \hat{i} - 203 \hat{j} + 42 \hat{k}$
 $v|_{t=5} = \sqrt{504^2 + 203^2 + 42^2} = 544.97 \text{ m/s}$
 $\vec{r} = (t^4 + 4t + 16) \hat{i} + (-\frac{8t^3}{3} - 3t - 20) \hat{j} + (4t^2 + 2t + 5) \hat{k}$
 $\vec{r}|_{t=5} = 629 \hat{i} - 368.33 \hat{j} + 115 \hat{k}$
 $r|_{t=5} = \sqrt{629^2 + 368.33^2 + 115^2} = 737.92 \text{ m}$
 $\Delta \vec{r} = (629 - (-16)) \hat{i} + (-368.33 - (-20)) \hat{j} + (115 - 5) \hat{k}$
 $= 645 \hat{i} - 348.33 \hat{j} + 110 \hat{k}$

(b)

$$\vec{a} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 300 & -80 & 8 \\ 504 & -203 & 42 \end{vmatrix}$$

$$= \hat{i} (-80 \times 42 + 8 \times 203) - \hat{j} (300 \times 42 - 8 \times 504) + \hat{k} (300 \times -203 + 80 \times 504)$$

$$= -1736 \hat{i} - 8568 \hat{j} - 20580 \hat{k}$$

$$|\vec{a} \times \vec{v}| = 22359.80$$

$$|\vec{a} \times \vec{v}| = \frac{v^3}{J}$$

$$\therefore J = \frac{(544.97)^3}{22359.80} = 7238.5 \text{ m}$$

$a_n = \frac{v^2}{J} = \frac{(544.97)^2}{7238.5}$
 $= 41.03 \text{ m/s}^2$
 $a^2 = a_t^2 + a_n^2$
 $\therefore a_t = \sqrt{a^2 - a_n^2}$
 $= \sqrt{310.59^2 - 41.03^2}$
 $= 307.87 \text{ m/s}^2$



x direction	y direction
$v_x = 32t - 4$	$v_y = 4$
Integrating	Integrating
$x = 16t^2 - 4t + c$	$y = 4t + c$
At $t = 3, x = 5$	At $t = 3, y = 12$
$\therefore c = -127$	$\therefore c = 0$
$\therefore x = 16t^2 - 4t - 127$	$y = 4t$
(4)	$\therefore t = \frac{y}{4}$
	Subst in (i)
$x = 16\left(\frac{y}{4}\right)^2 - 4\left(\frac{y}{4}\right) - 127$	
$x = y^2 - y - 127$	

(5) $r = 4m; a_t = 8m/s^2$
 $a = 10m/s^2$
 $a_n = \sqrt{a^2 - a_t^2}$
 $= \sqrt{10^2 - 8^2}$
 $= 6m/s^2$
 $a_n = \frac{v^2}{r}$
 $\therefore v = \sqrt{6 \times 4}$
 $= \underline{\underline{4.90m/s}}$

(6) $r = 400m$
 $u_A = 72 \times \frac{5}{18} = 20m/s$
 $v_c = 108 \times \frac{5}{18} = 30m/s$
 $s_{AC} = 300m; s_{AB} = 250m$
 $a_B = ?$
 $v^2 = u^2 + 2as$
 $30^2 = 20^2 + 2 \times a_c \cdot 300$
 $a_c = \frac{5}{6} m/s^2$
 Finding $v_B = ?$
 $v^2 = u^2 + 2as$
 $v_B^2 = 20^2 + 2 \times \frac{5}{6} \times 250$
 $v_B = 28.58m/s$
 $a_{nB} = \frac{v_B^2}{r} = \frac{28.58^2}{400} = 2.04m/s^2$
 $a_B^2 = a_{tB}^2 + a_{nB}^2$
 $\therefore a_B = \sqrt{\left(\frac{5}{6}\right)^2 + (2.04)^2}$
 $= 2.20m/s^2$



(7) $y = 10 + 0.4x^2$
 Diff. w.r.t 't'
 $v_y = 0.4(2x)v_x$
 $v_y = 0.8x v_x$ — (i)
 $v = 6 \text{ m/s}$

$\therefore \sqrt{v_x^2 + v_y^2} = 6$
 $\therefore v_x^2 + v_y^2 = 36$
 Using (i)
 $v_x^2 + (0.8x v_x)^2 = 36$

At $x = 3\text{m}$
 $v_x^2 + (2.4 v_x)^2 = 36$

$\therefore v_x = \sqrt{\frac{36}{1+(2.4)^2}}$
 $v_x = 2.308 \text{ m/s}$

Subst. in (i)
 $v_y = 0.8 \times 3 \times 2.308$
 $v_y = 5.538 \text{ m/s}$

(b) \therefore Const vel. $\therefore a_t = 0$
 $a_n = \frac{v^2}{r} = \frac{36}{r}$ — (i)

$\frac{dy}{dx} = 0.8x$; $\frac{d^2y}{dx^2} = 0.8$
 $f = \frac{[1 + (\frac{dy}{dx})^2]^{3/2}}{d^2y/dx^2}$
 $= \frac{[1 + (0.8x)^2]^{3/2}}{0.8}$

At $x = 3 = \frac{[1 + (0.8 \times 3)^2]^{3/2}}{0.8} = 21.97 \text{ m}$

$\therefore a_n = \frac{36}{f} = \frac{36}{21.97} = 1.639 \text{ m/s}^2$

$\therefore a = \sqrt{a_t^2 + a_n^2} = 1.639 \text{ m/s}^2$



(8) $\vec{a} = 10\hat{i} + 6t\hat{j}$ $t=0, v=0, x=0, y=0$
 at $t=4s, v=? , \vec{r}=? , \bar{r}=?$

$a_x = 10$
 Integrating
 $v_x = 10t + c$
 at $t=0, v_x=0$
 $\therefore c=0$

$a_y = 6t$
 Integrating
 $v_y = 3t^2 + c$
 At $t=0, v_y=0$
 $\therefore c=0$

$v_x = 10t$

$v_y = 3t^2$

Integrating
 $x = 5t^2 + c$
 at $t=0s, x=0$
 $\therefore c=0$

Integrating
 $y = t^3 + c$
 at $t=0, y=0$
 $\therefore c=0$

$x = 5t^2$

$y = t^3$

$\vec{a} = 10\hat{i} + 6t\hat{j}$
 $\vec{a}|_{t=4} = 10\hat{i} + 24\hat{j}$

$a|_{t=4} = \sqrt{10^2 + 24^2} = 26 \text{ m/s}^2$

$\vec{v} = 10t\hat{i} + 3t^2\hat{j}$
 $\vec{v}|_{t=4} = 40\hat{i} + 48\hat{j}$

$v|_{t=4} = \sqrt{40^2 + 48^2} = 62.682 \text{ m/s}$

$\vec{r} = 5t^2\hat{i} + t^3\hat{j}$
 $\vec{r}|_{t=4} = 80\hat{i} + 64\hat{j}$

$r|_{t=4} = \sqrt{80^2 + 64^2} = 102.45 \text{ m}$



$$\vec{a} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 & 24 & 0 \\ 40 & 48 & 0 \end{vmatrix}$$

$$= 0\hat{i} - 0\hat{j} + \hat{k}(480 - 960) \\ = -480\hat{k}$$

$$|\vec{a} \times \vec{v}| = v^3$$

$$480 = \frac{v^3}{62.482}$$

$$v = 508.187 \text{ m}$$



(9) $a_x = -5 \text{ m/s}^2$ $a_y = 2 \text{ m/s}^2$ at $t=0$, $v_x = 10 \cos 36.87^\circ$
 Integrating w.r.t t we get $v_y = 10 \sin 36.87^\circ$
 $\int t=8s = ?$

$v_x = -5t + c$ & $v_y = 2t + c$

at $t=0$ at $t=0$
 $v_x = 10 \cos 36.87^\circ$ $v_y = 10 \sin 36.87^\circ$

$\therefore c = 8$ $\therefore c = 6$

$\therefore v_x = -5t + 8$ $v_y = 2t + 6$

$\vec{a} = -5\hat{i} + 2\hat{j}$

$\therefore \vec{a}|_{t=8} = -5\hat{i} + 2\hat{j}$

$\vec{v} = (-5t + 8)\hat{i} + (2t + 6)\hat{j}$

$\vec{v}|_{t=8} = -32\hat{i} + 22\hat{j}$

$v|_{t=8} = \sqrt{32^2 + 22^2} = 38.833 \text{ m/s}$

$\vec{a} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 2 & 0 \\ -32 & 22 & 0 \end{vmatrix} = 0\hat{i} - 0\hat{j} + \hat{k}(-110 + 64)$
 $= -46\hat{k}$

$|\vec{a} \times \vec{v}| = 46$

$|\vec{a} \times \vec{v}| = \frac{v^3}{r}$

$46 = \frac{(38.833)^3}{r}$

$\therefore r = 1273.05 \text{ m}$



(10) $y = \frac{x^2}{8}$; $x = 2t^2 - 4t$ $v = ?$ at $x = 8m$
 $a = ?$

Subst (i) in (ii)

$$y = \frac{(2t^2 - 4t)^2}{8} = \frac{4t^4 - 16t^3 + 16t^2}{8}$$

$$\therefore y = \frac{t^4 - 4t^3 + 4t^2}{2} \quad \text{--- (iii)}$$

~~Differentiating~~ Differentiating (i) & (iii) we get

$$v_x = 4t - 4 \quad v_y = \frac{4t^3 - 12t^2 + 8t}{2} \quad \text{--- (iv)}$$

~~Differentiating~~ Differentiating w.r.t t

$$a_x = 4 \quad ; \quad a_y = \frac{12t^2 - 24t + 8}{2} \quad \text{--- (v)}$$

Put $x = 8$ in (i)

$$8 = 2t^2 - 4t$$

$$\therefore t = 3.236s$$

Sub in (iv), (v), (vi), (vii)

$$v_x = 8.944 \text{ m/s}$$

$$v_y = 17.887 \text{ m/s}$$

$$a_x = 4 \text{ m/s}^2$$

$$a_y = 27.998 \text{ m/s}^2$$

$$v = \sqrt{(8.944)^2 + (17.887)^2} = 19.998 \text{ m/s}$$

$$a = \sqrt{4^2 + (27.998)^2} = 28.282 \text{ m/s}^2$$



NOTE : distance $\xrightarrow{d/t}$ vel $\xrightarrow{d/t}$ tangential accel.

$$(11) s = 0.5t^3 + 3t \quad a = 10 \text{ m/s}^2 \quad f = ?$$

Differentiating
~~velocity~~ we get

$$v = 1.5t^2 + 3$$

Differentiating

$$a_t = 3t$$

$$\text{at } t = 2 \text{ s}; \quad a_t = 6 \text{ m/s}^2, \quad v = 9 \text{ m/s}$$

$$\therefore a = \sqrt{a_n^2 + a_t^2}$$

$$10 = \sqrt{a_n^2 + 6^2}$$

$$a_n = 8 \text{ m/s}^2$$

$$a_n = \frac{v^2}{r}$$

$$\therefore f = \frac{9^2}{8} = \underline{\underline{10.125 \text{ m}}}$$





12) at $t=0$, $x=0$, $f=10m$

$$s = 0.2t^3$$

$$a/t=3=?$$

Diff w.r.t t

$$v = 0.6t^2$$

Differ. w.r.t t

$$a_t = 1.2t$$

at $t=3s$

$$a_t = 3.6 \text{ m/s}^2 \quad (+i)$$

$$a_n = \frac{v^2}{f} = \frac{[0.6(3)]^2}{10} = 2.916 \text{ m/s}^2$$

$$a = \sqrt{a_n^2 + a_t^2} = \sqrt{3.6^2 + 2.916^2}$$

$$= 4.699 \text{ m/s}^2$$

